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MTH 341

Monday Homework

Proof of One of Euclid's Theorems

Theorem 1. (i) Every line segment has exactly one midpoint.

Proof. (i) Let C and D be midpoints of a line segment AB. By Axiom 4,

$$AC = BC \qquad \text{(i)}$$

Since D is also a midpoint, then

$$AD = DB \qquad \text{(ii)}$$

We have

$$AB = AB \qquad \text{(iii)}$$

And we know

$$AB = AC + CB$$
$$AB = AD + DB$$

From equation (iii)

$$AC + CB = AD + DB$$

From equation (i) and (ii) plug in the value of BC and DB, we get

$$AC + AC = AD + AD$$
$$2AC = 2AD$$

Divide by 2, we get

$$AC = AD$$

Both points are on the same line so both points will superimpose and D and C are exactly the same place.

Hence, the midpoint of these line segments is always unique.

Proof. (ii) Given ΔABC, find C' on $\xrightarrow[AC]{}$ such that $AB \cong AC'$.

Let D be the midpoint of AC'. By definition of midpoint,

$$AD \cong DC'$$

By construction,

$$BA \cong BC' \text{ (i)}$$

$$BD \cong BD \text{ (ii)}$$

By SSS,

$$\Delta DBA \cong \Delta DBC'.$$

We now have $\xrightarrow[BD]{}$ inside $\Delta ABC'$ and $\Delta ABD \cong C'BD$.

Therefore, $\xrightarrow[BD]{}$ bisects $\Delta ABC'$.

Now suppose that $\xrightarrow[BE]{}$ also bisects $\Delta ABC'$

$\xrightarrow[BE]{}$ is inside $\Delta ABC'$.

Therefore, $\xrightarrow[BE]{}$ meets AC' in a point E'.

$$BA \cong BC'$$

By definition of bisector,

$$\begin{aligned} \Delta EBA &\cong \Delta EBC \\ BE &\cong BE \end{aligned}.$$

By SAS,

$$\Delta EBA \cong \Delta EBC'$$

Therefore,

$$AE \cong EC'$$

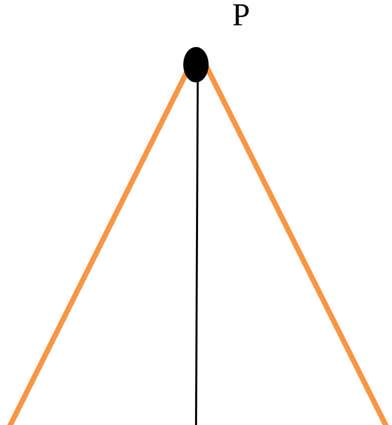
This makes E a midpoint of AC.

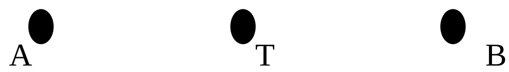
By uniqueness of midpoints,

$$E = D.$$

FINISH

Converse Theorem 3.2.8. If a point is equidistant from the endpoints of a line segemtn, then the point lies on the perpendicular bisector.





Proof. Given $\overline{AB}$ and a point P equidistant from its endpoints A and B:

$$AT = BT.$$

We need to show that the point P lies on the perpendicular bisector.

Let T be the midpoint of $\overline{AB}$. Connect the points P and T by the line $\overline{PT}$.

Consider the triangles ΔATP and ΔBTP. These triangles have the common side PT, the congruent sides AT and BT, and the congruent sides PA and PB. Hence, the triangles are congruent in accordance to the SSS postulate.

Therefore, $\angle ATP$ and $\angle BTP$ are congruent as the corresponding angles of the congruent triangles.

Since the sum of $\angle ATP$ and $\angle BTP$ is the straight angle ATP of $180°$, then $\angle ATP$ and $\angle BTP$ are half of $180°$, that is $90°$.

This means that $\overline{PT}$ is perpendicular to $\overline{AB}$.

Thus, the point P lies on the perpendicular bisector of the midpoint T of $\overline{AB}$. *QED*