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Can a Kite be a Triangle? Bidirectional Discourse and Student Inquiry in a Middle School Interactive Geometric Lesson

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Abstract - In this article we present a case story of a teaching episode that takes place in an accelerated middle school geometry class (N = 14) using Shape Makers, an interactive geometric curriculum. We use Tawfeeq's bidirectional discourse model and a modified version of Bonnstetterôs (1998) Inquiry Continuum to frame (1) the nature of the mathematical discourse, (2) the instructional decisions by the teacher, and (3) progressive conceptualizations of the students' geometric ideas elicited by a non-traditional representation of a kite in an interactive geometric curriculum.

Introduction

While the study of geometry has existed for many millennia, it is within the past 20 years that a shift has occurred in how geometry may be learned through computer-based interactive geometric software. Software programs like The Geometer's Sketchpad, Cabri Geometry, and Geogebra allow users to construct, on the computer, interactive representations of points, lines and circles that may be resized and shifted around onscreen through clicking and dragging actions. Moreover, interactive geometric software in the K-12 mathematics curriculum has been used at the elementary level (Battista, 2002; Yu & Barrett, 2002), middle school level (Jones, 2000; Yu, 2004; Yu, Barrett, & Presmeg, 2009), and the high school level (Hollebrands & Smith, 2009).

One such geometry curriculum, Shape Makers (Battista, 2003), uses interactive geometric objects in the form of various quadrilaterals (Figure 1) to help middle school students develop a dynamic understanding of the quadrilateral properties and relationships.

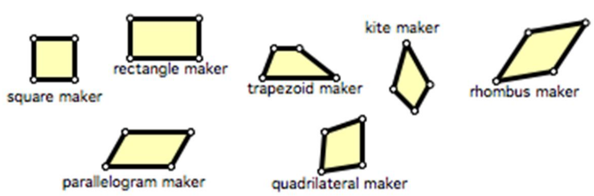

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Figure 1 - Quadrilateral Makers

For example, the Kite Maker is an interactive geometric shape that can be dragged and manipulated onscreen into various forms while preserving the defining qualities of a kite shape (Figure 2).

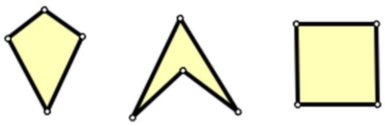

Figure 2 - Convex, non-convex and square forms of a kite

In this curriculum, students are led through a series of activities in which they explore visual, property-based, and relational characteristics of various quadrilaterals by dragging the shape makers' vertices to resize and morph the onscreen figures. In this article, we share a classroom episode in which middle school students explore the nature of mathematical definitions through an unintended degenerate case of the Kite Maker, which is in the form of a triangle (Figure 3).

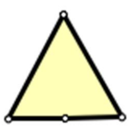

Figure 3 - Kite Maker in a triangle shape

Conceptual Considerations

To help frame the events in the classroom episode that follows, we present two models for classroom discourse, Tawfeeq's Bidirectional Discourse Model (BDM) and a modified version of Bonnstetter's (1998) Inquiry Continuum. The Bidirectional Discourse Model frames dialogue in the mathematics classroom in which the discursive nature between teacher and student are fluid and shift through four different levels. These levels are based on the role that the students and teacher have in the conversation. In figure 4, the first level of discourse is depicted by the innermost circle, *Teacher (responsible) to Student*. At this level, the mathematical dialogue is considered directional in which the teacher initiates and carries the responsibility for the conversation with the students, as in a classroom lecture. Moving outward, the second level of discourse, *Teacher to Student (responsible)*, the dialogue is initiated by the teacher, yet the student carries some responsibility for the mathematical discourse, as in a question and answer session. While the student shares responsibility for the

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discourse, this teacher-initiated dialogue is also considered directional. In the third level, *Student and Student*, the students are conversing about mathematical ideas, yet the ideas may only be conjectures and lack mathematical correctness, as in a small group investigative activity. However, at this level, open discursive interaction between students is considered bidirectional, as both conversational agents share equal responsibility in the discussion. In the outermost circle, level 4, *Student and Teacher* discourse is characterized by the students initiating the mathematical dialogue. This is considered bidirectional in that both students and teacher share equal responsibility in maintaining the mathematical conversation. Furthermore, at this level, the teacher may or may not know the outcome of the discussion, but assumes the responsibility to ensure mathematical correctness or consistency in the conversation. Finally, the concentric circles in the figure denote the embedded nature of the four levels of discourse, in that mathematical dialogue at level 4 may contain elements of levels 1, 2, or 3.

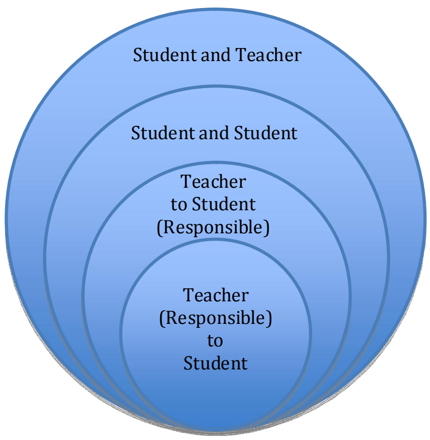

Figure 4. Directional and Bidirectional Discourse

A second model is a modified version of Bonnstatterôs (1998) Inquiry Continuum, which considers the domains of instruction, source of the mathematics topics, and source of the learning outcomes (Table 1). This model further illustrates the nature of mathematical discourse in the construction of mathematical knowledge. For example, the domain of instruction in the first column on the left is *Traditional Didactics*. Under this modality, the source of the mathematical topics, the discussion questions, and materials is the mathematics textbook.

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Furthermore, the teacher initiates the lessonǿs design, procedures, results and conclusions. In contrast, with the domain of instruction identified as *Student Initiated*, the source of the topic is the student. The students, with support of the teacher, generate the questions and conclusions. The teacher, with support of the students, chooses the course material, and the students guide the lesson's design, procedures, and results.

Modality	Traditional Didactic	Structured Inquiry	Guided Inquiry	Student Initiated	Open Inquiry
Topic	Mathematics Textbooks	Mathematics Textbooks	Teacher/ Student	Student	Student
Question	Mathematics Textbooks	Generated by Teachers	Teacher/ Student	Student/ Teacher	Student
Materials	Mathematics Textbooks	Mathematics Textbooks	Teacher	Teacher/ Student	Student/ Teacher
Procedures / Design	Teacher	Teacher	Teacher/ Student	Student/ Teacher	Student
Results	Teacher	Teacher/ Student	Teacher/ Student	Student	Student
Conclusion	Teacher	Teacher/ Student	Teacher/ Student	Student/ Teacher	Student

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Table 1. 5 Step Inquiry Domain Continuum That Develops Mathematical Maturity

We will use these models to describe the shift in mathematical dialogue as a result of explorations of quadrilaterals in an interactive geometric environment. This shift is away from teacher monologue, and towards

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classroom conversation in which rich mathematical concepts form the basis of bidirectional, student-initiated classroom conversation. Furthermore, the teacher and students were engaged in conversation because the mathematical context provided by the interactive geometric environment made the conversation necessary.

Interactive Geometry in the Classroom

The following episode was taken from a videotaped excerpt of a class discussion during a Shape Makers (2003) activity called "How are they the same?" The class in which the video was taken was a special section of an accelerated high school geometry class taught to a select group of fourteen middle school students ranging mostly from grades 6 through 8. In this particular activity, the students, working individually, had to list as many invariant properties of any shapes made with a particular Shape Maker as they morphed and reshaped the Shape Makers. For example, with the Kite Maker, commonly listed properties of any shapes made with the Kite Maker were: two pairs of congruent sides, at least one pair of congruent angles, and at least one line of symmetry. After a time of individual exploration, the students shared their observations to create a class list of properties for each shape. Each shape and its associated properties were written on poster board sized paper, and put at the front of the classroom for all the students to see. In most classes, the teacher of this lesson would conduct the activity as a form of *Structured Inquiry* (Table 1), because the topic and materials were based on the curricular text activity. The questions and lesson procedures were designed by the teachers in order to lead to a set of student-generated and teacher-guided results and conclusions. In the case of this lesson, the goal was the development of a comprehensive list of properties for each of the seven quadrilaterals. However, in this particular class, a number of students observed the Kite Maker in a degenerate form making a figure that looked like a triangle (Figure 3), prompting one student, Hank, to ask, ñIt is possible that the kite can be a triangle?ò

The teacher, initially unsure of the question, responded, "Ah... I dunno, what do you mean by the triangle thing?"

Hank followed up, ñIé Iôm looking for clarification, but is a triangle a valid kite?ò

~~Violet added, ñ[The computer] doesnôt say itôs not a kite when you make ité ò~~

Another student continued, ñYea. There are two different ways to make a triangle with a kite [maker].ò

Nick began to explain, ñI mean, you can make it so its three vertexes are lined up so technically itôsé ò

The teacher cut Nick off, ñIôm going to table that one, because it goes outside the bounds of this [activity]é Itôs an interesting questioné youôre talking about how you can take the kite makeré ò

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ñ...you can shift it into a triangle,ò finished Nick.

Initially, the teacher did not understand issue, and tried to "table" the questions raised by students. This degenerate case of a Kite Maker, in the shape of a triangle (Figure 3), was not in the original lesson plan. However, the number of students that seemed engaged in the issue prompted the teacher to pursue a discussion of the issue. Seeking clarification, the teacher asked, ñYou shift [the Kite Maker] into a triangle, youôre asking is that thing really a kite?ò

Hank added, ñBecause you can do more stuff with the computer that maybe you canôt do with [pencil and paper].ò

ñThatôs a great question. What the heck, letôs talk about it,ò conceded the teacher.

This change of course marks an initial shift from a *Structured Inquiry* lesson intended by the teacher, to a *Student Initiated* lesson as the teacher responded to the students' mathematical question. Without a particular direction in mind for this aspect of the lesson, the teacher turned to the class. "Thoughts?"

~~Kellen and Kristie raised their hands. ñWhat are your thoughts? Kellen?ò~~

~~Kellen pointed to the 6 quadrilateral property sheets that had been generated, ñWere talking about quadrilaterals here. All these shapes are quadrilaterals, but a triangle is noté .ò~~

~~ñKristie?ò redirected the teacher.~~

Kristie, speaking to the teacher, began, ñOn the first day you said, told us the Shape Maker Rule was ôthat any shape made by a shape maker is that shapeé ~~ôò~~

~~ñI did say that.ò~~

~~She continued, ñé~~ and so any shape made with a Kite Maker is a kite shapeé ò

ñI did say that.ò Responded the teacher.

She concluded, ñé so wouldnôt that make, that because you made the triangle with the kite [maker], that means that the triangle is a type of kite?ò

Nick added, ñThere are only 3 sides [and 4 line segments]ò

Violet said, ñWhen we were messing around [with the seven quadrilateral makers] on the first day, all the shapesé ah, the only one that ever said that this is not what ever shape maker it is, was one time, the Quadrilateral Maker, and that was when you made the hour glass shape.ò (Figure 5).

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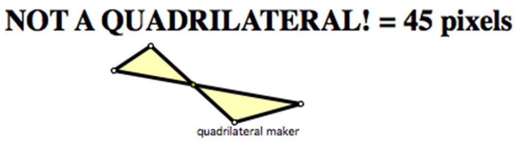

Figure 5 - Hourglass quadrilateral

Note how the dialogue had shifted to a bidirectional dialogue between Level 4 (*Student and Teacher*) and Level 3 (*Student and Students*). However, at this point in the conversation, much of the students' comments were speculation and conjecture. Although, Kristie tried to justify her reasoning with the "Shape Maker Rule" previously established by the teacher, the mathematical justification was visually-holistic in nature, and based on the actions of the interactive Kite Maker. Creating a visual point of focus to the discussion, the teacher sketched the degenerate example of the Kite Maker on the board, labeling the four vertices ABCD (Figure 6) ñSo the issue is, looking at this, Iôve got figure ABCD, where C is on the sameé ò In creating a visual point of focus, the students began to reason about the mathematics using the properties and component parts of figure ABCD.

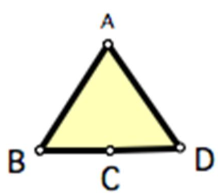

Figure 6 - Kite ABCD?

Kellen, pointing to the shape ABCD the teacher had drawn, added, ñBut triangles' angles have to add up to 180, but if we have three 60ôs and a 180, that goes over the limité so we canôt have that itôs a triangle.ò

Nick continued, ñI see what [Kellen's] going até technically 4 angles isnôt totally correct because it is possible, so it could be a vertex and not an angleé ò

Amber asked, while pointing to the teacherôs drawing of ABCD on the board, ñDoes a triangle have only three vertices? Because that shape has 4 vertices, doesnôt it?ò

Redirecting the conversation between Nick and Amber, the teacher asked, ñSo youôre [Nick] saying a triangle has three vertices, and she [Amber] is asking, does this shape have 4 vertices? Why do you [Amber] think it has 4 vertices?ò

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Gesturing towards the bottom half of ABCD, along line B to C to D, Amber clarified her idea, ñBecause the line where B and C, and C and Dé converge at C, so C is where two lines come together.ò

ñSo the issue is this. If I cover up this point C, ñ the teacher said while covering point C with his hand, ñhow many vertices [in the figure] do we have?ò

ñThree,ò answered the class in unison.

ñThree, but when I do that,ò continued the teacher as he removed his hand revealing point C, ñHow many vertices do we have?ò

ñFour,ò said the class.

Nick asked, ñIs C a vertex, or a point. Couldnôt [C] just be a point?ò

Responding to Nick's question the teacher clarified, ñWell, thatôs a good point. We havenôt clearly defined vertices. A vertex is é the intersection of two line segments, at their endpoints.ò

Violet concluded aloud, ñSo [C] is a vertex.ò

In the previous dialogue, the teacher assumed two roles of responsibility in clarifying the students' ideas. First, in covering and uncovering point C, the teacher sought to clarify Amber's point regarding "B and C, and C and D" to the class in a rhetorical questioning manner. Second, while responding to Nick, the teacher adds clarity to the concept of "vertices" by simply stating a definition. However, to maintain the bidirectional dialogue and the *Student Initiated* nature of the classroom lesson, the teacher turned to an idea shared earlier in the class discussion, ñAmber, could youé . I think youôre going down a trail of thoughté . you said, [that] a triangle has three vertices. Well, youôre saying, doesnôt [shape ABCD] have 4? So whatôs the point of your statement?ò

ñIt has 4 vertices, so it canôt be triangle,ò answered Amber.

ñSo then [ABCD] has to be what?ò Asked the teacher.

ñA kite, or a quadrilateral,ò concluded Amber.

~~At this point, the teacher directly addressed the conversational tangent, ñThis is important math here in this discussion because thereôs something going on with this shape [ABCD] that you guys kind of got the ball rollingé that Iôll entertain this for a few more minutes, because I donôt want it to spiral around, around and aroundé .ò~~ The teacher continued the discussion of figure ABCD. Using Amberôs conclusion he said, ñLetôs go with Amberôs point. She says look, how many vertices does it have? Four. Right? How many line segments is it made of? Four. But here is the question, is that thing [ABCD] a kite?ò The question momentarily hung.

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Referring back to the list of properties of a kite the class generated through their explorations with the Kite Maker, the teacher continued by applying each of the listed properties to figure ABCD, ñNow, here is what we have. Does this, this shape ABCD, does it ascribe to everything that weöve talked about so far? Does it have two pairs of equal sides?ò

In unison the class responded, ñYesò

ñWhat are the two pairs of equal sides?ò the teacher asked.

Violet responded, ñAD and AB.ò

ñOk, so segment AD is congruent to segment AB.ò The teacher repeated while drawing congruence markings on the corresponding sides. ñAnd what is the other pair of equal sides?ò

Violet again responded, ñDC and BC.ò

ñOK, segment DC is congruent to segment BCé ò Again, the teacher drew on the picture. He then continued, ñDoes [ABCD] have at least one line of symmetry?ò

A few students, pointing out the obvious, said, ñYes!ò

ñWhere?ò asked the teacher.

ñA to C,ò responded the class.

ñA to C. So, AC is a line of symmetry,ò repeated the teacher as he drew in the line of symmetry. ñDoes it have at lease one pair of equal angles?ò

ñYes,ò a few students answered.

ñWhich one?ò asked the teacher.

ñAngle D and angle B,ò said Carter.

ñOk, so the measure of angle D is equal to the measure of angle B.ò The teacher drew in the congruent markings on the corresponding angles. ñOk, fair enough, right? Ahé Is it made up of 4 line segments?ò

ñYes,ò said the class.

ñé And 4 angles?ò the teacher continued.

ñYes,ò said the class.

At this point the teacher slowed down the pace of the class discussion, ñIs it made up of 4 angles?ò he asked in a questioning tone.

The class responded emphatically, ñYes!ò

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Clarifying, Kellen added, ñC is one angle.ò

ñWhat type of angle is C?ò asked the teacher.

ñStraight,ò was the simultaneous response by a number of students throughout the class.

ñ[Angle C is] a straight angle, isnœt it?ò The teacher wanted to make sure that the class understood that while collinear with B and D, point C formed the vertex of a straight angle. For this portion of the discussion, the discourse shifted to Level 2 (*Teacher to student with student responsible*), as the teacher guided the students by applying the Kite's property list to shape ABCD. ~~However, without the teacher definitively identifying the true nature of ABCD, the students were still uncertain as to whether or not shape ABCD was a kite or a triangle.~~ However, the mathematical conversation became more abstract as the students begin to reason about the issues by attempting to use mathematical definitions. The discussion that follows shifted back to Level 4 (*Student and Teacher*) as characterized by the bidirectional nature of the discourse.

Nick tried to bring clarity to the issue, ñI think.... ~~one easy way to settle this argument now, and still be correcté~~ a kite [maker] can make a shape that looks like a triangle, but by definition is not a triangleé .ò

~~Hank, quickly agreed, ñSure.ò~~

Kellen had a different way to look at it, ñWe know itœs not a triangle, because we proved it has too many angles.ò

From the back of the room Travis asserted, ñI can prove that a kite is not a legitimate triangle.ò

The teacher stepped in to moderate the class conversation, ñO.K, we are going to go to you in a minute, Travis, when we make sense of what [Nick] saidé ò Turning towards Nick the teacher said, ñSay that again.ò

Nick repeated, ñA kite [maker] can make a shape that looks like a triangle, but by definition is not a triangleé ò

The teacher repeated his statement to make sure the class understood the point, ña kite [maker] can make a shape that looks like a triangle, but by definition is not a triangleé ò

Violet asked, ñwhat is the definition of a triangle then?ò

The teacher clarified, ñthree vertices and three sidesé even though we havenœt formally discussed this in class, all of us can agree that a triangle has three vertices and three sides.ò

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Travis then stated his point, ñA kite cannot be a legitimate triangle because properties of a triangle and properties of a quadrilateral conflicté because the insides of the angles of a quadrilateral, when added equal 360, but it will only equal 180 with a triangle.ò

ñLet me ask you this Travis, this shape right here ABCD,ò asked the teacher pointing to the figure on the board, ñdo you think that shape right here is a kite?ò

ñYes, because C is 180 degrees,ò answered Travis.

ñOK, so ABCD is a kite? What do you guys think? Chris?ò

The class responded in unison, ñYes.ò

Playing devilôs advocate, the teacher added, ñOK, so is ABCD a triangle?ò

This time the class responded, ñNo.ò

Violet added, ñNot by definition.ò

The teacher continued, ñNot by definition, but ité ò

ñé looks like one [a triangle],ò said the class finishing his statement.

The teacher then summarized the whole discussion by pointing out that given the current list of properties for a kite, shape ABCD was a kite, in the form of a triangle. This conclusion may be inconsistent with one's visual intuition, and definitions of polygons that do not allow for three adjacent collinear vertices. However, many textbook definitions of quadrilaterals and polygons omit qualifying statements like, no three adjacent collinear vertices. In the absence of such qualifying statements, and given the mathematical context, the class's conclusion was indeed mathematically valid. Realizing this, and to conclude the class's discussion, the teacher stated, "This was an interesting discussion because it helps us think through these [listed] properties in what I would consider a very unique case of a kite that is really cool for discussion within the context of a mathematics discussion. But within the context of the real world, is moot or irrelevant, because it looks like a triangle, so let's just call it a triangle."

Reflections

In the classroom episode presented, a non-traditional, yet mathematically rich discussion transpired due to interplay between the contexts created in the interactive geometric environment and the teacher's instructional decisions. First, Hankôs comment, "you can do more stuff with the computer that maybe you can't do with [pencil and paper]," reflects the notion that certain mathematical actions and phenomena may only be physically represented

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in an interactive geometric environment. For example, referring back to Figure 2, the images of the convex and non-convex kites are easily represented in static media like a textbook page, or whiteboard drawing. On the other hand, the action of physically dragging a vertex, to go back and forth between the convex and non-convex versions of a kite is particular to interactive geometry. Furthermore, it was precisely this seemingly trivial, and routine action that created the degenerate kite shape eliciting the question, "is a triangle a valid kite?"

Second, there were instructional decisions made by the teacher worth noting. Initially, when the question, "is a triangle a valid kite?" was asked, a number of options existed that would have significantly changed the nature of the mathematics lesson. One option was to maintain his initial inclination to "table" the idea as it was "outside the bounds of" the *Structured Inquiry* activity, and continue with the lesson's intended trajectory. Another option would have been to specify that in a quadrilateral, no three adjacent vertices could be collinear, and dismiss the students' question based on a more specific definition. In this case, the teacher chose to shift the modality of the lesson to *Student Initiated* as the students took the primary roles in generating the topic, questions, results and conclusions. The role of the teacher was to mediate the lesson discussion.

Another important instructional decision by the teacher was to maintain the bidirectional flow of discourse between *Student and Student*, and *Teacher and Student*. Throughout the lesson the teacher maintained the bidirectional flow through clarifying students' comments, and redirecting the students' comments between each other. Furthermore, when the teacher did ask questions of the students, the nature of the questions were prompts for the students to clarify their ideas. This bidirectional discourse advanced the mathematical conversations as new ideas, conjectures, and mathematical statements were added to the classroom discourse.

The teacher used the students' comments, ideas, and statements as a discursive context in which a central conclusion was formed, in this case, that ABCD was a kite, based on properties, yet in the shape of a triangle. Moreover, the importance was not in the mathematical veracity of whether or not ABCD was a triangle, but rather in the class discussion that helped the students "think through these properties" of a kite in order to draw a satisfactory conclusion.

One limitation of the teaching episode is that the teacher did not discuss with the students how and why some definitions of polygon and quadrilateral do include qualifying phrases like, no three consecutive collinear vertices, in order to dismiss degenerative cases as experienced in this lesson. More so, there was no discussion about the arbitrary nature of mathematical definitions as necessitated by a particular mathematical context. This

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teaching episode also illustrates a potential limitation of interactive geometric software. The interactive nature of the onscreen geometric representations allows users to experience examples, iterations, and types of shapes that may or may not conform to traditional notions of those shapes, in this case a kite in the shape of a triangle. However, these limitations are mentioned to bring awareness to the need for teachers of mathematics to familiarize themselves not only to the flexibility of interactive geometric shapes, but also to the possible instructional opportunities that may arise when students are given the chance to explore in an interactive geometric environment.

Conclusion

We present this classroom episode as an example of a mathematics lesson consisting of engaging and mathematically rich discourse that may be elicited by an interactive geometric environment. Also, we use this episode as a contrast to middle school mathematics lessons that may emphasize procedures and applications of procedures, with little to no attempt to make mathematical connections. ~~(Hiebert et. al., 2003).~~ Finally, to recommend teachers to ask open-ended questions is not enough. Rather, appropriate mathematical contexts need to be created to provide an appropriate discursive context for a more rigorous approach to asking open-ended questions. As illustrated in this episode, teachers should encourage their students to compare and contrast other students' responses, to verbally critique the questions of other students and the teacher, and to view such interaction as necessary and very important.

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